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Refining the Results of the Electronic Theory of Reflection of Metals

Konstantin Ludanov

Department No 5, Institute for Renewable Energy of NASU, Kiev, Ukraine

Email address:
k.i.ludanov@ukr.net

Abstract: The article provides an accurate analytical approximation of the expressions of the known results of the electronic theory of reflection of metals, i.e. for the spectral and integrated reflectivity of $\rho_{\lambda\perp}$ and $\bar{\rho}_\perp$ (Drude and Hagen-Rubens formulas, as well as the formulas of Ashkinass, Foote, Eckert and Drake, which are fragments of power series). Compact closed expressions for $\rho_{\lambda\perp}(\lambda)$ and $\bar{\rho}_\perp(\lambda)$ are obtained, and published experimental data on the reflectivity of the polished metal surface in new coordinate systems are processed and analyzed: $(ln\rho_{\lambda\perp})^{-2} \sim \lambda$ and $ln(\bar{\rho}_\perp)^{-1} \sim$ T. It turned out that the experimental data on $\rho_{\lambda\perp}(\lambda)$ in the new coordinates clearly "lie" on the straight lines, which in the general case do not pass through the origin, which required introducing into the expression $\rho_{\lambda\perp}(\lambda)$ a new parameter $- \lambda_o$, taking into account the difference between the "optical" conductivity $\sigma_o(\omega)$ from the electrical σ_e, where λ_o constant specific to each of the metal (for example, Ag, and Al: $\lambda_o>0$, for Ni and W: $\lambda_o= 0$, for Au and Cu: $\lambda_o<0$). In obtaining the final formula for $\rho_{\lambda\perp}(\lambda)$ a new mathematical derivation scheme was used, starting with an analysis of a pair of equivalent expressions of the complex refractive index of the metal — much more justified and brief.

Keywords: Electronic Theory of Reflection of Metals, Normal Spectral Reflectivity, Optical Conductivity, Approximation of Power Series, Processing of Experimental Data

1. Introduction

The modern theory of the optical properties of metals is based on complex electronic, kinetic, statistical, and quantum models. Historically, the first theory of reflection was developed by Drude, [1], based on the model of "free" electrons. However, a significant discrepancy between the calculation results $\varepsilon_{\lambda\perp}$ according to the Drude formula and experience led to the fact that, firstly, they began to refine it (Hagen and Rubens) by introducing the second and third members of the series, and secondly, the second version of the electron model (Drude-Zener model [1, 2]), in which a new concept of the "optical conductivity" of the metal σ_o was introduced, taking into account the well-known fact of the dependence of the conductivity on the wavelength of light λ. But even this was not enough to explain the very significant difference between the theoretical calculation and experimental data $\varepsilon_{\lambda\perp}$ in the near and middle part of the infrared region of the spectrum. Roberts [3, 4] suggested that valence electrons are divided into two types (s and d), each of

which creates its own optical conductivity, depending on the wavelength differently. On this basis, Edwards et al. [5, 6] created a method for calculating the "boundary" optical properties of good conductors (very complex, including a number of multi-constant approximations) − for most metals in the infrared range of the spectrum, which is currently widely used in practice [7].

The electronic model of the optical properties of metals adjoins the skin-layer model [8], obtained on the basis of kinetic theory. However, the solution obtained in its framework for $\varepsilon_{\lambda\perp}$ is also found in the form of the first terms of the power series, which leads to a low accuracy in calculating the reflectivity.

Standing apart is the metal reflection model developed by Dmitriev on the basis of statistical theory [9]. Although it well explains some optical effects (for example, the inversion of the spectral emissivity of metals at the X-point), but it has not received distribution, since it is not connected with electronic models, at least as a special or limiting case.

Finally, a quantum theory of the optical properties of

metals was developed [8, 10], which explains the absorption of their surface not only by transferring radiation energy to the ion lattice by field-accelerated conduction electrons, but also due to quantum transitions in the visible and ultraviolet regions. However, the contribution of quantum effects in the infrared range was insignificant (less than 10%), [11].

2. Literature Review

Drude model. The electronic model of metal reflection according to Drude [1] suggests that all valence electrons of the metal are free, that is, practically unconnected with the neighboring ions of the crystal lattice and are "electron gas". As a result of solving the problem of the interaction of the electromagnetic field with the "electron gas" of a metal, Drude obtained the first (linear) approximation for the spectral emissivity $\varepsilon_{\lambda\perp}$ in the direction normal to the surface (according to Kirchhoff's law, it is identically equal to absorption capacity: $\varepsilon_\lambda \equiv \alpha_\lambda$) $\varepsilon_{\lambda\perp} \cong 0{,}365\sqrt{(r_e/\lambda)}$. The main advantage of this formula to Drude is that it does not contain unknown constants: it was possible to connect the theoretically determined quantity $\varepsilon_{\lambda\perp}$ with only one quantity - the electrical resistivity r_e (Ohm·mm^2/m) − a property well studied for most metals and accurately measured over a wide temperature range. The Drude model allows one to calculate $\varepsilon_{\lambda\perp}$ in the $\lambda > 10\mu m$ region with reasonable accuracy [12]; however, it is very inaccurate in the middle and near infrared regions. This caused attempts to refine it by Hagen and Rubens [9], who received a second, and then the third term of the power series to express the variance $\varepsilon_{\lambda\perp} = \varphi(\lambda)$:

$$\varepsilon_{\lambda\perp} = 0{,}365\sqrt{(r_e/\lambda)} - 0{,}0667(r_e/\lambda) + 0{,}0091\sqrt{(r_e/\lambda)^3} \qquad (1)$$

However, the inclusion of three members of the series did not lead to satisfactory agreement between the calculation by the formula and the experimental data on average, and even more so in the near infrared range [12].

Drude-Zener model. This model takes into account the well-known fact that the electrical conductivity of a metal for direct current differs from its conductivity with respect to high-frequency alternating current [1, 2]. In this case, conduction electrons with a certain inertia lag behind the field oscillations in phase and the metal conductivity included in Ohm's differential law (j = σE) becomes complex. Drude and Zener wrote down the complex conductivity σ in the form $\sigma = \sigma_e \cdot exp(i\cdot\omega/\gamma)$ and obtained in scalar form the expression for "optical conductivity": $\sigma_o = \sigma_e/(1 + \omega^2/\gamma^2)$, where γ − decrement of attenuation. But even this introduction did not significantly improve the accuracy of the calculation of $\varepsilon_{\lambda\perp}$.

Roberts model. It also appeared in the framework of the electronic reflection model [3], and, in particular, develops the Drude – Zener interpretation in terms of the optical conductivity of the metal. However, in the Roberts model, valence electrons are no longer "free", but "quasi-free". Moreover, not one, but two types of valence electrons (s and d) are introduced with different relaxation times τ_i. And each one creates its own optical conductivity:

$$\sigma_{1o} = \sigma_e/(1 + i\lambda_1/\lambda), \sigma_{2o} = \sigma_e/(1 + i\cdot\lambda_2/\lambda) \qquad (2)$$

In a scalar form, the total optical conductivity is expressed:

$$\sigma_o = \sigma_e/[1 + (\lambda_1/\lambda)^2] + \sigma_e/[1 + (\lambda_2/\lambda)^2] + \sigma_\infty \qquad (3)$$

Later Roberts [4] also introduced the third group of carriers with conductivity σ_3, and also took into account corrections for resonance phenomena. Therefore, the method of calculating the optical properties of metals (the method of Edwards et al. [5, 6]), created on this basis, presented, for example, in [7] turned out to be very complicated, because included several empirical constants, which in the calculation must first be determined on the basis of experimental data.

3. Formulation of the Problem

Analysis of publications shows that the electronic theory of reflection of metals, although it began to be developed first, has not yet been completed. First, the expressions for the spectral and integrated emissivity are fragments of the power series (Drude, Hagen-Rubens, etc. formulas for $\varepsilon_{\lambda\perp}$ and the Ashkinass, Eckert-Drake, and Fout formulas for $\overline{\varepsilon_\perp}$). And every fragment of a series very inaccurately reflects its sum, which generally represents a certain function that decomposes into this series (this point of view was first expressed by Euler [13]). Thus, there is the problem of finding a function that would accurately approximate the sum of a series by its fragment. A good basis for such a search is given by the Padé approximation [13, 14]. However, it involves approximation only in the form of rational fractions of polynomials, and the form of the chosen function is often dictated by the physical meaning of the problem.

Secondly, there is a certain arbitrariness in the "construction" of the formulas for the optical conductivity of a metal, which is characterized by the presence of a number of "theoretically substantiated" expressions for complex conductivity: 1) $\sigma_o = \sigma_e exp(i\omega/\gamma)$ and 2) $\sigma_o = \sigma_e/(1 + i\omega\tau)$, also for the conductivity σ_o in scalar form: 3) $\sigma_o = \sigma_e/[1 + (\omega/\gamma)^2]$ and 4) $\sigma_o = \sigma_e/[(1 + \omega^2\tau^2)^{0,5} - \omega\tau]$, [8]. Thus, in this case too, there is the problem of finding a new, more correct expression for the optical conductivity of the metals $\sigma(\omega)$. And finally, thirdly, the situation that has developed with the results for integrated reflectivity is similar to the situation with spectral results for $\rho_{\lambda\perp}$. Indeed, the well-known expressions for the $\overline{\varepsilon_\perp}$ also represent fragments of power series [9, 15].

In this regard, the aim of this work is to refine the results of the electronic theory of reflection of metals, which will solve the above problems and allow us to create the foundations of a new (more accurate and correct) method for calculating "boundary" optical properties of metals (ρ, ε, a) for the infrared region of the spectrum.

4. Research Results

4.1. Derivation of the Closed Expression $\rho_{\lambda\perp}$ for Metals

It is known that the whole complex of optical and

electrophysical properties of the metals [8, 10] is concentrated in two equivalent expressions for the complex refractive index N:

$$N = n - i \cdot k \equiv \sqrt{(э - i \cdot 2\sigma/\nu)} \qquad (4)$$

The spectral reflectivity of an optically smooth metal surface is expressed through N as follows:

$$\rho_{\lambda\perp} = |N - 1|^2 / |N + 1|^2 \qquad (5)$$

If we substitute the first expression for N into this formula and square the corresponding modules, then we can obtain the well-known Beer's formula:

$$\rho_{\lambda\perp} = [(n - 1)^2 + k^2]/[(n + 1)^2 + k^2] \qquad (6)$$

For metals (i.e., very good conductors) it was established [8, 10] that the prevailing part of the energy of the field penetrating through their surface when irradiated with light has a magnetic nature (for dielectrics $\sigma = 0$ and this ratio is 50% to 50%). Let us analyze the expression of this ratio:

$$\mu \ \overline{H^2}/(э \ \overline{E^2}) = \sqrt{[1 + 4(\sigma/э\nu)^2]} \qquad (7)$$

Since for the metal the left-hand side significantly exceeds unity, it is quite possible to write that $\sigma/\nu \gg э$. And in this case, the quantity э under the root sign in the second expression N can simply be neglected.

The expression for the complex refractive index N (taking into account that $\sqrt{(-2i)} = 1 - i$) takes the following form:

$$N \cong \sqrt{(-2i\sigma/\nu)} = (1 - i)\sqrt{(\sigma/\nu)} \qquad (8)$$

Comparing the result with a second expression for N, we conclude that under the condition $\sigma/\nu \gg э$ we have $n \cong k \cong \sqrt{(\sigma/\nu)}$. And the Bera formula for metal in this case will look like this:

$$\rho_{\lambda\perp} = (2n^2 - 2n + 1)/(2n^2 + 2n + 1) \qquad (9)$$

Since this expression is quite complex, only the first terms of its expansion in a series in negative powers of n are used [16] and, since for metals n > 1, this expansion takes the following form:

$$\rho_{\lambda\perp} = 1 - 2n^{-1} + 2n^{-2} - n^{-3} + \ldots \qquad (10)$$

Drude, Hagen and Rubens used, respectively, two, three, and then four members of this decomposition to obtain their results (it is given above). Moreover, fragments of power series were also obtained, while compact closed expressions are important for applications, which approximate the sum of the series quite accurately and reflect the physics of the phenomenon.

Therefore, as early as in [17], the author analyzed the expansion structure for $\rho_{\lambda\perp}(n)$ and found that the best approximation for the sum of the reduced series (10) is the exponent:

$$exp(-2/n) = 1 - 2n^{-1} + 2n^{-2} - 1{,}33n^{-3} + \ldots, \qquad (11)$$

the expansion of which also gives an alternating power series that differs from the expansion in a series of Beer's formula only by the value of the coefficient in the fourth term. Using this fact, we obtain a new closed expression of spectral reflectivity:

$$\rho_{\lambda\perp} = exp(-2/n) \qquad (12)$$

Substituting instead of the quantity n its expression found above, we obtain:

$$\rho_{\lambda\perp} = exp[-2\sqrt{(\nu/\sigma)}] \qquad (13)$$

And if we substitute the resistivity and wavelength instead of conductivity and frequency, then we can finally write:

$$\rho_{\lambda\perp} = exp[-2\sqrt{(r_e/30\lambda)}] = exp[-0{,}365\sqrt{(r_e/\lambda)}] \qquad (14)$$

The expression obtained here is not only sufficiently accurate and compact, but also corresponds to the physics of the reflection phenomenon, since it correctly reflects the asymptotic behavior at $\lambda \to 0$, in contrast to the Drude formulas and others. However, when trying to present experimental data from directories in the coordinate system $(ln\rho_{\lambda\perp})^{-2} \sim \lambda$, where they were supposed to "lie" on the straight lines passing through the origin, problems arose. It turned out that the experimental points can indeed be described by linear dependencies, but the lines obtained for various metals in the general case do not pass through the origin. Therefore, the resulting expression (14) also requires refinement.

4.2. A New Expression for $\rho_{\lambda\perp}$ Metal, Taking into Account the Optical Conductivity $\sigma_o(\omega)$

The presence of several expressions σ_o both in complex and in scalar form indicates that this question has not been finally resolved. An analysis of experimental data shows that in most cases the optical conductivity is less than the electrical conductivity for direct current, which is why in the Drude-Zener formula $\sigma_o \leq \sigma_e$. However, in some cases [8], for example, for alkali metals, $\sigma_o \geq \sigma_e$. That is why formulas of the Drude-Zener type for representing optical conductivity were not acceptable.

First, the generalized expression for optical conductivity should be based on the phenomenological Maxwell relations [8], that is, the polarization current should also be taken into account:

$$j = \sigma E + \alpha(\partial E/\partial t) \qquad (15)$$

Usually, this expression of the sum of the conduction and polarization currents is presented in the form of Ohm's law in differential form:

$$j = j(\sigma) + j(\alpha) = \sigma_o E, \qquad (16)$$

and for optical conductivity in scalar form, we can write:

$$\sigma_o(\omega) = \sigma_e + F(\gamma, \alpha, \omega), \qquad (17)$$

which on the contrary gives only $\sigma_o \geq \sigma_e$, which is also unacceptable.

Therefore, a different approach to solving this problem is needed. If we consider that σ_o is not very different from σ_e, then a fairly accurate estimate of $\sigma_o(\omega)$ can be obtained by expanding its expression in the Taylor series in the variable ω, and assuming that the resulting series is fast converging, use only the first two terms:

$$\sigma_o(\omega) = \sigma_o(0) + (\partial\sigma_o/\partial\omega)\cdot\omega + \ldots \cong \sigma_e(1 + f\cdot\omega), \qquad (18)$$

where $\sigma_o(0)=\sigma_e$, and regarding the coefficient $f = (\partial\sigma_o/\partial\omega)/\sigma_e$ we can definitely say that it depends on the damping decrement γ of valence electrons and on the polarizability of the lattice ions α:

$$(\partial\sigma_o/\partial\omega)/\sigma_e = f(\gamma,\alpha) \qquad (19)$$

Potentially here it can be either positive ($\sigma_o>\sigma_e$), so negative ($\sigma_o<\sigma_e$), or equal to zero ($\sigma_o = \sigma_e$). Substituting this result in the expression $\rho_{\lambda\perp}$, we obtain the final formula:

$$\rho_{\lambda\perp}= exp\{-\sqrt{[r_e/\lambda\cdot(1+f\cdot\omega)\cdot C_1]}\}=exp\{-\sqrt{[r_e/(\lambda+\lambda_o)\cdot C_1]}\}, \quad (20)$$

where σ_o is the only new parameter reflecting the influence of the damping decrement of the conduction electrons γ and the polarizability of the crystal lattice ions on α in the framework of the concept of "optical conductivity" (here $\sigma_o/\nu \rightarrow \sigma_e\cdot(\lambda + \lambda_o)$), $C_1 = 7,5$.

Thus, here, in the framework of the electronic theory of reflection of metals, the author obtained a new expression for the normal reflectivity of the surface of metals - taking into account the presence of optical conductivity of their surface:

$$\rho_{\lambda\perp}= exp\{-0,365\sqrt{[r_e/(\lambda+\lambda_o)]}\}, \qquad (21)$$

Processing of experimental data for $\rho_{\lambda\perp}$ [15, 18, 19] obtained with normal reflection of a polished metal surface in the coordinates $(ln\rho_{\lambda\perp})^{-2} \sim \lambda$ showed that they very clearly obey the linear dependence. In most cases, these straight lines do not pass through the origin and cut off individual λ_o values for each metal on the ordinate axis, which convincingly testifies to the exact closed expression for $\rho_{\lambda\perp}$ found above, taking into account the new relation for the optical conductivity σ_o.

The data obtained for some metals (Me) are presented in the tables below.

Table 1. Data obtained for some metals (Me).

Me	Al	Ag	Ni	W	Au	Cu
$\lambda_0\mu m$	+5,8	+2,3	0	0	− 0,6	−1,0

Me	Fe	Rn	Ti	Pt	Zn	Cr
$\lambda_0\mu m$	−1,8	−1,9	−2,0	−2,2	−2,5	−4,6

If, as the lower boundary λ_{min} of the spectral range in which the electronic theory of metal reflection works, it is quite reasonable to use λ_X (the inversion point for the dispersion of the reflectivity of the surface: for tungsten $\lambda_X =$

1.28 µm and for rhenium $\lambda_X = 1.0$ µm), then as the upper boundary of the range λ_{max} it is quite obvious − the intersection point of the curve $\rho_{\lambda\perp}= exp\{-0,365\sqrt{[r_e/(\lambda+\lambda_o)]}\}$ with line $(\rho_{\lambda\perp})_{max} = const$ for the region $\lambda_{max}< \lambda < \propto$. The values of the coordinates of the intersection points (boundaries of neighboring regions λ_{max}) for some metals, established as a result of processing the experimental data, are presented in Table 2.

Table 2. A result of processing the experimental data.

Me	Al	Ag	Ni	W	Au	Cu
$\lambda_{max},\mu m$	3,0	1,0	2,0	2,0	2,0	1,0
$(\rho_{\lambda\perp})_{max}$	>0,987	>0,99	0,98	0,97	>0,987	0,975

Me	Fe	Rn	Ti	Pt	Zn	Cr
$\lambda_{max},\mu m$	3,0	2,0	6,0	3,0	3,0	6,0
$(\rho_{\lambda\perp})_{max}$	0,90	0,96	0,92	0,96	0,98	0,98

4.3. Integral Reflectivity of Metal $\bar{\rho_\perp}(T)$

The integral emissivity $\bar{\varepsilon_\perp}$ of the polished metal surface in the normal direction is obtained by integrating the product of the expression $\varepsilon_{\lambda\perp}$ and the Planck function for the radiation of the blackbody over the entire spectrum. Using G-functions, Ashkinass integrated the linear Drude approximation [16] and established:

$$\bar{\varepsilon_\perp}\cong 0,576\sqrt{(r_eT)}, \qquad (22)$$

a result that is suitable only for an approximate forecast, and not for an accurate calculation. Eckert and Drake refined the linear expression by integrating the second term [20] and obtained:

$$\bar{\varepsilon_\perp}= 0,576\sqrt{(r_eT)} - 0,124(r_eT) \qquad (23)$$

Foot [9], by analogy with Eckert and Drake, got the third term in the expression above and recalculated the coefficients of the series taking into account the new values of both constants in the Planck formula − c_1 and c_2:

$$\bar{\varepsilon_\perp}= 0,578\sqrt{(r_eT)} - 0,179(r_eT) + 0,044(r_eT) \qquad (24)$$

The author of this article earlier [17] for expression (23) also found an exact and compact approximation (taking into account $\varepsilon_\perp \equiv 1 - \rho_\perp$) in the form of the Newton binomial:

$$\bar{\rho_\perp}= \{1 -\sqrt{[(r_eT)/48]}\}^4 = 1 -\sqrt{[(r_eT)/3]} + (r_eT)/8 - \ldots \qquad (25)$$

To verify the accuracy of the proposed approximation, it is logical to present the experimental data on the dependence of the integrated reflection of the polished metal surface on $\bar{\rho_\perp}$ in the coordinates $(\bar{\rho_\perp})^{0,25} \sim \sqrt{(T)}$. However, as was shown above (in Section 4.2), the assumptions adopted in this integration are not entirely correct, since they "work" only in the range $\lambda_X< \lambda \leq \lambda_{max}$. In the region $\lambda < \lambda_X$, quantum laws already work, and for $\lambda > \lambda_{max}$ the reflectivity of the metal surface is practically independent of the wavelength and $\bar{\rho_\perp}$ = const. Therefore, in this case, the expression for $\rho_\perp(\lambda)$ does not integrate in quadratures. In this situation, the most optimal solution is approximation by a closed expression

within the exponent. This approximation in [18] was proposed by Helfgott in the form: $\overline{\varepsilon}_\perp \cong 1 - exp(-a \cdot T)$, where a (K^{-1}) is an individual value for each metal.

By the way, within the framework of the Ashkinass approach, it is also possible to obtain an exponential expression for $\overline{\rho}_\perp(T)$ based on the theory of approximating the sum of a series [14] in the following form:

$$\overline{\rho}_\perp = exp\{-\sqrt{[(r_e T)/3]}\} = 1 - 0.5773\sqrt{(r_e T)} +..., \quad (26)$$

and try to process the data on the reflection of metals in the coordinates $(\overline{\rho}_\perp)^{0,25} \sim \sqrt{(T)}$. However, it is known that for many metals electrical resistivity r_e (Ohm·cm) under the condition $T > 300$ K is directly proportional to the absolute temperature: $r_e(T) = (r_e)_o \cdot (T/273)$ [16]. In this case for $\overline{\rho}_\perp$ there should be a linear dependence in the coordinates $ln(\overline{\rho}_\perp)^{-1} \sim T$. Indeed, analysis of published data [12] shows that, for example, the integrated reflectivity of nickel and platinum in the temperature range 500 – 1300 K accurately obeys a linear dependence, and the resulting straight lines again do not pass through the origin, and experimental data are generalized by an exponential expression of the form:

$$\overline{\rho}_\perp = exp[-a \cdot (T - T_o)], \quad (27)$$

where taking into account (25) $a \sim \sqrt{(r_e)_o}$. For example, for Ni: $T_o = 80$ K, $a = 1.5 \cdot 10^{-4}$ (K^{-1}) and for Pt: $T_o = 194$ K, $a = 1.34 \cdot 10^{-4}$ (K^{-1}).

5. Discussion of Results

An analysis of the experimental data on the normal spectral reflectivity of the polished metal surface within the framework of the new coordinate system showed that the electronic theory of reflection "works" only in a rather narrow range of wavelengths: $\lambda_X < \lambda < \lambda_{max}$ of the middle and far infrared ranges, and accordingly a very small change in spectral reflectance: $\rho_{max} > \rho_{\lambda\perp} > 2/3$. The presence of a maximum value ρ_{max} of a smaller unit follows from the analysis of experimental data (reference [19]) and does not "fit" into the representations of the electronic model, since $lim\rho \rightarrow 1$ follows from it for $\lambda \rightarrow \infty$. As the lower boundary of this range ($\rho = 2/3$), the result is also used not from the electronic model, but from the Dmitriev statistical model. If for the lower boundary $\rho = 2/3$ there is a well-defined interpretation of Dmitriev (this is the reflectivity at the X-point), then for the upper boundary ρ_{max} there is no justified interpretation.

The exponential expression obtained in this work for $\rho_{\lambda\perp}$, firstly, is much more accurate than the Drude and Hagen-Rubens formulas approximates the Beer's formula for metal (9), since it is known [13] that the sum of the discarded terms of the alternating power series is less than its first discarded terms, and the accuracy of the approximation used can be estimated as follows: $\Delta < 1/3n^3$ (where for $\rho \geq 2/3$ from the Beer's formula $n \geq 5$), secondly, it includes the only new parameter λ_o, individual for each metal (and λ_o can be positive, negative and equal to zero).

It is one parameter λ_o, and not two (λ_1 and λ_2), like Roberts [3], that takes into account the influence of an electromagnetic wave (light incident on the surface) on the electrons of the metal: on valence (quasi-free) electrons - through the damping decrement γ, and to bound electrons - through the polarizability of ions α.

Therefore, it can be argued that the "one-electron" scheme in the framework of the electronic reflection model (in which all conduction electrons behave identically and are quasi-free) quite accurately and correctly describes the dispersion of the normal reflectivity of metals, and the use of the "two-electron" Roberts scheme [3, 4] completely unreasonable. Thus, the rather complicated method for calculating the optical characteristics of metals developed by Edwards et al. [5, 6] based on the Roberts scheme may well be replaced by a very simple and reasonably justified calculation method $\rho_{\lambda\perp}$ based on exact closed analytical solutions obtained in this paper.

An estimate of the "gap" between λ_X and λ_{min} (the lower boundary of the approximation range and the point X) using the example of tungsten, for which $\lambda_X = 1.28$ μm and $\lambda_{min} = 2.3$ μm, showed that it is small and is $\delta\lambda = 2.3 - 1.3 = 1$ μm. Thus, it can be assumed here that this "gap" is small for other metals and that most of the infrared range (up to λ_{max}) is accurately described by the dependences found in the work.

An analysis of the experimental data [19] for the normal spectral reflectivity of platinum at room temperature and at T = 1125 K showed that λ_o depends on temperature rather weakly (in the first case $\lambda_o = 2.2$ μm, and in the second $\lambda_o = 1.4$ μm), decreases with increasing temperature, and the form of the formula $\rho_{\lambda\perp} = \varphi(T)$ is determined mainly by the temperature dependence of the electrical resistivity. We can assume a similar dependence $\lambda_o(T)$ for other metals.

6. Findings

A compact closed analytical expression of the spectral reflectivity of a polished metal surface in the normal direction with a new ratio for "optical conductivity", obtained for the first time in this work, not only qualitatively, but also quantitatively correctly describes the published experimental data and can become the basis of a new calculation method "boundary "optical properties of metals, provided that a data bank is created for only one parameter - $\lambda_o(T)$.

The obtained expression describes with sufficient accuracy the dispersion of the spectral reflectivity of metals in the infrared region of the spectrum (because its domain is $\lambda_X < \lambda < \lambda_{max}$, and the range is $2/3 < \rho < \rho_{max}$). The data from this area is usually used in pyrometric measurements to determine the temperature of a metal surface in engineering and scientific research, so the results are very relevant. For the purpose of pyrometry, it is normal optical characteristics ($\varepsilon_\perp$ or $\rho_\perp$, [20]) that are important, since for the metal surface only radiation in the normal direction is not polarized.

Attempts by Ashkinass, Eckert-Drake, and Foote to obtain integral characteristics for the reflectivity of metals by

integrating the Drude formula in the full radiation range ($0 < \lambda < \infty$) are obviously in accurate, because they do not take into account the fact that the laws of the electronic theory of metal reflection "work" only in a very limited part of the spectrum: $\lambda_X < \lambda < \lambda_{max}$.

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