The Role of Polarization and Spontaneous Emission in Lasing on the Basis of Maxwell's Equations

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Abstract: This research paper mainly aims to investigate the effect of polarization and susceptibility in amplifying light and producing laser using Maxwell equations, which consist of polarization term, and using quantited density matrix. It was shown that the polarization and the susceptibility both contributors to amplification. Strictly speaking, both stimulated photons and spontaneous photons, which in the direction of the field increase polarization.

Keywords: Polarization, Spontaneous Emission, Maxwell Equations

1. Introduction

Physics plays an important role in our day life, for it provides us with the tools that promote and generate technologies. For example, the phenomenon of stimulated emission (According to this process, if a molecule finds itself in a higher energy level and is struck by a photon, it may decay to a lower energy state, if the photon has exactly the energy corresponding to the energy difference between the two states. When decaying to the lower energy state, the molecule emits a photon that has exactly the same energy of the first one, while the first photon continues its journey unaffected, happy to have 'stimulated' the molecule to de-excite.) is used to produce laser, which is used in a wide variety of applications: to cut and weld metal for building ships, automobiles, and railroad cars; to perform surgery; to put on fabulous light shows; to gather forensic evidence; and

to measure the distance from earth to the moon accurately. Lasers are also used to make very small features in materials. These small features are critical to the electronics, aerospace, medical, renewable energy, automobile, and other industries [1, 2]. Laser is an acronym for Light Amplification by Stimulated Emission of Radiation. The acronym gives a clue to the process by which this amazing light is generated, but not what it is. It may, however, be called a device which produces an amazing 'beam' of light with some unique properties. The term 'laser' is now interchangeably used to mean the source and the beam. Lasers were first demonstrated in the 1950s in the microwave region of the electromagnetic spectrum, and this subset of lasers was called a MASER (Microwave Amplified by Stimulated Emission of Radiation). In the estimated emission a photon incident on excited atom forces it to emit another photon the two photons can also stimulate two atoms to emit four

photons, which in turn stimulate other four atoms and as a result light can be amplified [3].

Properties of laser beams

Laser radiation is characterized by an extremely high degree of monochromaticity, coherence, directionality, and brightness. We now consider these properties in some detail.

i. Monochromaticity: the laser light is nearly monochromatic. In reality, no light is perfectly monochromatic, i.e., it is not characterized by a single wavelength λ or frequency ν but instead; it is characterized by spread in frequency $\Delta\nu$ about the central frequency (or $\Delta\lambda$ in case of wavelength λ).

The monochromaticity of a light is defined by $\dfrac{\Delta\nu}{\nu}$.

For perfect monochromaticity $\Delta\nu = 0$, which is not attainable in practice, but the value of $\Delta\nu$ is much smaller for lasers compared to ordinary light. For ordinary light $\Delta\nu$ is of the order 10^{10} Hz, whereas for a laser, it of the order of 500 Hz. Thus, laser light is highly monochromatic compared to ordinary light [4, 5].

ii. Coherence: Coherence is one of the striking properties of laser, over the conventional sources, which makes them useful for several scientific and technological applications. The basic meaning of coherence is that all the waves in the laser beam remain spatially and temporarily in the same phase. Photons generated through stimulated emission are in phase with the stimulating photons. There are two types of coherence: time coherence is when two beams are emitted from the same source at the same instance, and then superposed onto the same point. And space coherence is when two beams are emitted from the same source, but at different times, and are then superposed onto one point. If they do or not produce an interference fringe after superposition, they are considered as coherence or non-coherent, respectively, of which the interference scale is denoted by the coherence time τ_c and coherence length L_c respectively.

iii. Directionality: one of the most striking properties of laser is its directionality, that is, its output is in the form of an almost parallel beam. Owing to its directional nature it can carry energy and data to very long distances for remote diagnosis and communication purposes. In constant, conventional light sources emit radiation isotropically; therefore, very small amount of energy can be collected using lens. Beam of an ideal laser beam is perfectly parallel, and its diameter at the exit window should be same to that after traveling very long distances, although in reality, it is impossible to achieve.

Brightness: lasers are more intense and brighter sources compared to other conventional sources such as the sun. A 1 mW He-Ne laser, which is highly directional low divergence laser source, is brighter than the sun, which is emitting radiation isotropically. Brightness is defined as power emitted per unit area per unit solid angle. in the particular case of 1 mW He-Ne laser with 3.2×10^{-5} rad beam divergence and 0.2 mm spot diameter at exit window the solid angle $(\pi(3.2\times10^{-5})^2)$ is 3.2×10^{-9} sr and spot area $(\pi(2\times10^{-4})^2)$ at the exit window is $1.3\times10^{-7}\,m^2$. Thus the brightness of the beam is given by $((1\times10^{-3})/(1\times10^{-7}\,m^2\times3.2\times10^{-9}\,sr)) = 2.4\times10^{12}\,W/m^{-2}sr^{-1}$, which is almost 10^6 times brighter than the sun $(1.3\times10^6\,W/m^{-2}sr^{-1})$.

These above properties allow lasers to deliver very high power density within a very narrow wavelength band onto targets – even on ones located at a considerable distance [6, 7, 8].

Lasers have numerous applications and can be considered as one of the main technological outputs of quantum physics, and have become excellent and inevitable tools in fields such as material processing (drilling, cutting, welding, heat treatment etc.), production of electronic components, nuclear science (isotope separation, laser fusion, counting of atoms etc), photochemical processes, ranging and tracking, spectroscopy, medical science (especially in surgery), laser communication (communication via the use laser is a fast emerging technology that aims to transmits data using very high intensity laser), defence, pollution monitoring etc. [9, 10, 11, 12, 13].

To increase laser application in our day life is it important to search for new ways to produce laser different attempts were made to search for new lasing mechanisms [14, 15]. Some of them used special relativity [16, 17], while others utilize the relation between amplification factor and conductivity [18]. One of the new physical quantities, which are affecting the production of the laser, is the polarizability and susceptibility. In this wave, the role of these quantities is discussed besides the effect of the spontaneous emission on the production of the laser beam. It was shown that type of susceptibility affect amplification of light. This is done in sections (2), (3) and (4) are devoted to discussion and can ciusion.

2. Wave Equation

All electromagnetic phenomena can be said to follow from Maxwell's equations.

In 1864, Maxwell proposed a Dynamical Theory of the Electromagnetic Field and thus unified the experimental researches of over a century through a set of equations known as Maxwell's equations. These equations were verified by Hertz in 1887 in a brilliant sequence of demonstrations [19]. It is now generally accepted that all electromagnetic phenomena are governed by Maxwell's equations. For a charge-free homogeneous, isotropic dielectric, Maxwell's equations simplify to

$$\nabla.E = 0 \qquad (1)$$

$$\nabla.H = 0 \qquad (2)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \tag{3}$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \tag{4}$$

Where ε and μ represent the dielectric permittivity and the magnetic permeability of the medium and E and H represent the electric field and magnetic field, respectively.

For most dielectrics, the magnetic permeability of the medium is almost equal to that of vacuum, i.e.,

$$\mu = \mu_o = 4\pi \times 10^{-7} \, NC^{-2} s^2$$

Equations (1, 2, 3 and 4) are linear, meaning that superimposed solutions still solve the equations. It is worth noting that this linearity is not always true, in particular in materials the fields may not be simply additive. Even in vacuum, quantum mechanics would predict nonlinearity, because virtual pairs of charged particles created by photons in the field can scatter other photons in the field.

3. Cavity Wave Equation

The general wave equation as started in Eq. (5)

$$\nabla^2 E(r,t) - \mu o \frac{\partial E(r,t)}{\partial t} - \mu\varepsilon \frac{\partial^2 E(r.t)}{\partial t^2} = -\mu \frac{\partial^2 p}{\partial t^2} \tag{5}$$

In the cavity, the wave will be directed along one direction say (z). The spontaneous emission random and since we expect an excess photon, we assume that photon will be the only direction and polarized.

The electric field along z is

$$E(Z,t) = E_0 e^{i(wt-kZ)} + c.c \tag{6}$$

Substitute Eq. (6) in Eq. (5) yields

$$\frac{\partial^2 E_0}{\partial z^2} - 2ik\frac{\partial E_0}{\partial z} - k^2 E_0 - i\sigma\mu w_0 E - w^2 \mu\varepsilon_0 E = x\mu\varepsilon_0 w^2 E_0 \tag{7}$$

for rotating approximation

$$\frac{\partial^2 E_0}{\partial z^2} \ll \frac{\partial E_0}{\partial z} \quad and \quad \mu\varepsilon w^2 \ll x\mu\varepsilon_0 w^2 \tag{8}$$

and for non-conducting medium $\sigma = 0$ Eq. (7) becomes

$$-2ik\frac{\partial E_0}{\partial z} - k^2 E_0 = x\mu\varepsilon_0 w^2 E_0 \tag{9}$$

The general solution for the wave cavity Eq. (9) is

$$\frac{\partial E_0}{E_0} = \frac{1}{2ik}(x\mu\varepsilon_0 w^2 - k^2)dz \tag{10}$$

$$\int \frac{\partial E_0}{E_0} = \int \frac{1}{2ik}(x\mu\varepsilon_0 w^2 - k^2)dz \tag{11}$$

$$E_0 = e^{\frac{k}{2} - x\mu\frac{\varepsilon_0 w^2}{2k}} \tag{12}$$

The rate of transition from the upper-level b to the lower one a is given by

$$\frac{dp_b}{dt} = \frac{1}{2}\Omega_R sin\Omega_R \tag{13}$$

for small values of Ω_R one get

$$\frac{dp_b}{dt} = \frac{1}{2}\Omega_R{}^2 = \frac{1}{2}\left|\frac{P_A}{\hbar}\right|^2 \mathcal{E}_0^2 \tag{14}$$

Where $P_A = \langle|er|\rangle$ is the average dipole moment per unit atom, which is related dipole moment per unit volumes by the relation

$$P = NP_A \tag{15}$$

Where N is the number of atoms per unit volume, therefore the probability of transition from existed state can be written as

$$\frac{dP_b}{dt} = \frac{\langle|er|\rangle\mathcal{E}_0^2}{2\hbar^2 N^2} = \frac{\mathcal{E}_0^2}{2\hbar^2 N^2}P^2 \tag{16}$$

The intensity of photons inside the active medium is given by;

$$E_m^2 = |E_{in} + xE_{in}|^2 \tag{17}$$

Where E_{in} represents the initial spontaneous emission electric field intensity amplitude and $xE_{in}=P$ stand for the excess intensity due to the interaction of the medium.

Therefore, the total number of the photons is given by

$$n = E_{in}^2 + 2PE_{in} + P^2 = E_{in}^2 + \left(\frac{2}{x} + 1\right)P^2 \tag{18}$$

Therefore, one expects the term $\left(\frac{2}{x} + 1\right)P^2$ to stand for the number of photons, which are generated spontaneous emission, and through stimulated emission at any time.

The expression, which is obtained by using Maxwell equation, is complete conformity with the expression at the rate of spontaneous and stimulates transition from upper level to the lower one, which is obtained quantum mechanically. This means that the X and P are proportional to the rate of spontaneous and stimulated transition. Therefore, one can split X into two components in the form

$$P = (X_{st} + X_{sp})E \tag{19}$$

Thus, it is well known that the stimulated photon is in the same direction as the incident one the stimulated susceptibility (X_{st}) is directed along the axis of the active medium. While spontaneous susceptibility (X_{sp}), which represents random photons, can be split into two components one is to the axis of the active medium and other is perpendicular to this axis i.e.

$$X_{sp} = X_{sp1} + iX_{sp2} \tag{20}$$

Therefore;

$$P = (X_{st} + X_{sp1})E + iX_{sp2}E \tag{21}$$

Substituting Eq. (21) in Eq. (12) yields

$$E_0 = expi\left(\frac{k}{2} - \frac{\mu\varepsilon_0 w^2[(X_{st}+X_{sp1})+iX_{sp2}]}{2k}\right)z \tag{22}$$

$$E_0 = expi \left(\frac{k}{2} - \frac{c_0[(X_{st}+X_{sp1})+iX_{sp2}]}{2k} \right) z \quad (23)$$

Hence $c_0 = \mu\varepsilon_0 w^2$

Thus since the direction of emitted photons are random. Thus, one can split wave number to parallel and perpendicular one, the wave number is complex i.e.

$$K = K_1 + iK_2 \text{ and } \overline{K} = K_1 - iK_2 \quad (24)$$

Hence

$$E_0 = expi \left(\frac{k}{2} - \frac{c_0[(X_{st}+X_{sp1})+iX_{sp2}]\overline{K}}{2k\overline{K}} \right) z \quad (25)$$

$$E_0 = expi \left(\frac{k}{2} - \frac{c_0[(X_{st}+X_{sp1})+iX_{sp2}][K_1-iK_2]}{2(K_1^2+K_2^2)} \right) z \quad (26)$$

$$E_0 = expi \left(\frac{k}{2} - \frac{c_0[(X_{st}+X_{sp1})+iK_1X_{sp2}-ik_2(X_{st}+X_{sp1})-K_2X_{sp2}]}{2(K_1^2+K_2^2)} \right) z \quad (27)$$

$$E_0 = exp \left(\frac{c_0 k_2(X_{st}+X_{sp1})}{2(K_1^2+K_2^2)} - \frac{c_0 k_1 X_{sp1}}{2(K_1^2+K_2^2)} \right) z \, x \, expi \left(\frac{k}{2} - c_0 \left(\frac{k_1(X_{st}+X_{sp1})+k_2 x_{sp2}}{2(K_1^2+K_2^2)} \right) \right) z \quad (28)$$

But the intensity is given by

$$I = |E|^2 = E_0^2 exp \left(\frac{2c_0 K_2(X_{st}+X_{sp1})}{2(K_1^2+K_2^2)} - \frac{2c_0 k_1 X_{sp1}}{2(K_1^2+K_2^2)} \right) z \quad (29)$$

On the other hand

$$I = I_0 e^{\alpha z} \quad (30)$$

Hence

$$-\alpha = \left(\left(\frac{2c_0 K_2(X_{st}+X_{sp1})}{2(K_1^2+K_2^2)} - \frac{2c_0 k_1 X_{sp1}}{2(K_1^2+K_2^2)} \right) \right) \quad (31)$$

Therefore, amplification is proportional to X_{st} and X_{st1} which indicate that stimulated emission and spontaneous emission photons in the direction of the active medium axis increases amplification while X_{st2} which represents spontaneous photons in the direction perpendicular to active medium axis decreases amplification.

4. Discussion

By using rotating approximation the solution of Maxwell equation Eq. (5) is given by Eq. (12). The fact that the probability of transition is dependent on polarization (see Eq. (14)) and this fact contributes to amplification. This motivates to express the emitted photons in terms of susceptibility according to Eq. (17). Furthermore, Eq. (18) shows that emitted photons depend on polarization. It has been found that the amplification factor as shown above in Eq. (31) comes as a result of splitting wave number to the real and imaginary part which accounts for photons electric components parallel and perpendicular to the lasing direction. However, Eq. (31) shows that both stimulated photons and spontaneous one, which are coherent with stimulated ones,

contribute to lasing.

5. Conclusion

The significant term that reflects the effect of the medium in amplifying radiation, as found in the current study, is polarization. Theoretically speaking, basing on both quantum mechanics and electromagnetic theory, polarization has been found to be related to the probability of transition.

Polarization, on the one hand, enables us to study the effect of spontaneous photon emission on amplification. It was proved that spontaneous photons emitting in the direction of the stimulated photon increases amplification. On the other hand, amplification factor is derived from splitting wave number to imaginary and real part.

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